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A New Set of Fifth and Sixth-Order Vented-Box Loudspeaker System Alignments using a Loudspeaker-Enclosure Matching Filter: Part I

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ABSTRACT

A new vented-box loudspeaker system is introduced that can be tuned to provide a pre-determined frequency-response shape over a fairly wide and continuous range of box volumes. A conventional high-pass filter only allows the system to be tuned to give a particular frequency-response shape if the box volume is correct. The conventional filter can be either isolated (i.e. buffered by the amplifier) or non-isolated (i.e. between the amplifier and loudspeaker). The latter could be a passive filter that interacts directly with the complex load-impedance of the loudspeaker. Consequently, the two cases require different box volumes. A new current-feedback filter is introduced that can provide a continuous range of alignments from isolated to non-isolated.

0 INTRODUCTION

The bass-reflex vent extends the low-frequency response of the system whilst reducing diaphragm excursion and hence also distortion at around the box Helmholtz resonance f_B. The filter reduces diaphragm excursion below the box resonance, where it would otherwise be greater than if the box were sealed. One of the problems in implementing a vented-box design, especially in the case of portable equipment, is that the volume of the box is often dictated by the constraints of the industrial/mechanical design of the product. Using conventional alignments, the chances of the available box volume V_B being the optimum for the desired frequency-response shape are very slim, although an elegant set of alignments by Keele [1] went some way towards resolving this. In this paper, the Butterworth shape is used as an example since it is fairly common in loudspeaker design. The 2nd-order Loudspeaker Enclosure Matching Filter (LEMF) is introduced first, as it is generally better than the 1st-order version, which is just included here for completeness. Firstly, the 2nd-order LEMF has a steeper roll-off and is thus more effective at reducing low-frequency excursion. Secondly, it has a much greater range of solutions than the 1st-order version. As far as implementation goes, there isn't much difference in complexity between them.

Since the great pioneering work of Neville Thiele [2] and Richard Small [3], the process of designing speaker systems has been simplified by using just six parameters to completely characterise a driver (at least at low frequencies), known as the Thiele-Small parameters, and then using alignment tables or charts to generate the system parameter values. In fact, several easy-to-use proprietary software applications are now available that can calculate these parameters thus enabling a complete system design to be produced very quickly without any mathematics. The six Thiele-Small parameters are R_E, f_S, Q_{ES}, Q_{MS}, V_{AS} & S_D where

R_E is the electrical dc resistance of the voice coil (Ω)
f_S is the mechanical resonant frequency in free space (Hz)
Q_{ES} is the electrical Q due to R_E
Q_{MS} is the mechanical Q due to mechanical viscosity R_{MS}
Q_{TS} is the total Q given by

$$Q_{TS} = \frac{Q_{ES}\,Q_{MS}}{Q_{ES} + Q_{MS}}$$

V_{AS} is the volume of air that exhibits the same compliance C_{MS} as the suspension, where compliance is the inverse of stiffness (m^3)
S_D is the effective surface area of the diaphragm (m^3)

V_{AS} is the reference volume used for calculating the box volume. The required box volume V_B can be expressed as a dimensionless ratio V_B/V_{AS} as shown in Table. 1. This is the inverse of the compliance ratio that is conventionally shown in alignment tables, but is more intuitive for gauging the relative box size. The cut-off frequency f_3 is also expressed as a dimensionless ratio f_3/f_S, using the mechanical resonant-frequency f_S as the reference.

Relative box size V_B/V_{AS}	Discrete Butterworth solutions using conventional filters	f_3/f_S	Power lift	Continuous Butterworth solutions using LEMF
2.64	Type 1 non-isolated 2nd order filter	0.49	3.5 dB	2nd order LEMF (2.64 to 1.00)
1.89	Non-isolated 1st order filter	0.62	Nil	1st order LEMF (1.89 to 1.00)
1.36	Class III isolated 2nd order filter	1.00	Nil	
1.00	Isolated 1st order filter & Class II isolated 2nd order filter	1.00	Nil	
0.94	Type 3 non-isolated 2nd order filter	2.05	Nil	No Butterworth solutions using either LEMF or conventional filter (0.94 to 0.37)
0.71	No filter (4th-order system)	1.00	Nil	
0.37	Class I isolated 2nd order filter	1.00	5.7 dB	
0.22	Type 2 non-isolated 2nd order filter	1.00	10 dB	2nd order LEMF (0.37 to 0.22)
0.00				

Table 1. Summery of conventional and LEMF Butterworth vented-box system alignments

The table gives a summary of the conventional Butterworth solutions together with the new LEMF ones. In order to avoid confusion, the Thiele 6th-order isolated filter alignments are referred to by their original classification of Class I, II and III and the 6th-order non-isolated alignments are referred to here as Type 1, 2 and 3.

Due to the complexity of the equations, the B6 LEMF alignments presented here do not take into account absorption, leakage or vent losses (Q_A, Q_L, Q_B respectively). Essentially they are loss-less Thiele alignments. Small suggested that Q_A, Q_L and Q_B could be combined together as an equivalent Q_L value. It will be shown in Part 2 that these enclosure losses can be accounted for by increasing V_B together with a much smaller increase in some of the other parameters. Some correction factors will then be derived for a Q_L value of 7.

Mechanical loss (Q_{MS}) is also omitted from the model shown in Fig.7. However, Q_{MS} is usually much greater than Q_{ES} for most drivers. Therefore, setting the Q_{TS} values in the alignment tables to be equal to the ideal Q_{ES} values calculated for the loss-less model results in only very small errors in most cases. The tables provide initial parameter values that can be fine-tuned during simulation.

1 2ND-ORDER FILTERS

1.1 2nd-Order Isolated Filter (3 solutions)

A.N. Thiele [2] recognised that incorporating an isolated high-pass filter prior to the input of the power amplifier, as shown in Fig. 1, could do much to solve the low-frequency excursion problem associated with vented boxes.

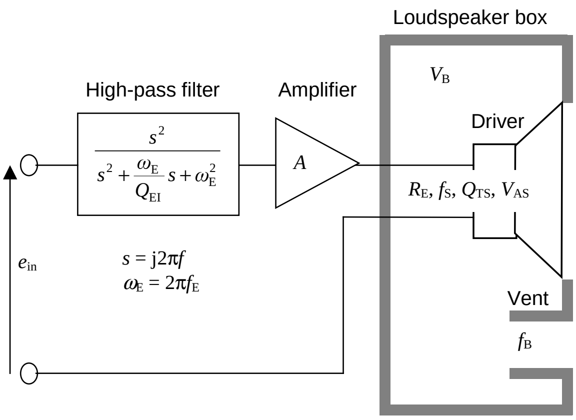

Fig. 1. Vented-box system with 2nd-order isolated filter

He showed that the best results are obtained by designing the filter in conjunction with the loudspeaker in order to produce a particular 6th-order frequency-response shape. He also provided alignment tables in order to engineer such responses and his original Butterworth (B6) alignments are reproduced here as a subset of the Type 2 LEMF alignments given in Table 2.

An added bonus of using an isolated 2nd-order filter is that there are three solutions for any given frequency-response shape thus allowing three possible box sizes. In the case of the Butterworth solutions, the largest box size (Class III B6) is the most efficient and the smallest (Class I B6) is the least efficient, requiring some assistance from the amplifier in the form of a 5.7 dB peak in the filter's response. This peak is derived from the filter's Q_{EN} value of 1.932. The values of the compliance ratio V_{AS}/V_B are 2.732, 1.000 and 0.732 for Classes I, II and III respectively. All three B6 solutions have a cut-off frequency f_3 that is equal to f_S. The suspension resonant-frequency f_S provides a useful reference for the box and filter resonant-frequencies, f_B and f_E respectively, in alignment tables, as well as for the cut-off frequency f_3.

1.2 2nd-Order Non-Isolated Filter (3 solutions)

If we place a passive filter between the amplifier and loudspeaker, as shown in Fig. 2, we get three solutions for any particular frequency-response shape. However, due to the complex interaction between the filter and the speaker's input-impedance Z, these solutions are different from those for an isolated filter.

D. R. von Recklinghausen [4] suggested the use of such a filter for extending the low-frequency response of a driver in a sealed box and provided alignment design tables. However, alignments have been provided here for a non-isolated filter with a vented box, as shown in the first row of table 2. In the case of the Butterworth solutions, the values of V_{AS}/V_B are 0.379, 4.464 and 1.067 for Types 1, 2 and 3 respectively. The cut-off frequencies are $0.487f_S$, $1.000f_S$ and $2.052f_S$ respectively.

Type 1 provides just over an extra octave of low-frequency extension using a large enclosure, whilst Type 2 allows the use of a very small box, albeit with a 10 dB peak in the filter's response. The latter represents a tenfold increase in input power at the cut-off frequency. Type 3 has a high cut-off frequency with a medium sized enclosure, but minimises diaphragm displacement and input power at low frequencies.

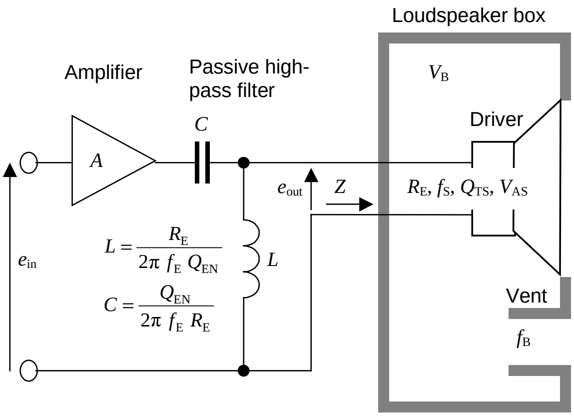

Figure 2. Vented-box system with passive 2nd-order non-isolated filter

If the inductor were replaced with a transformer of suitable winding inductance, this type of passive filter could be used in 100V-line PA loudspeaker systems with the added advantage that the capacitor would reduce the risk of magnetic saturation within the transformer core. Another somewhat specialised application could be the matching of the loudspeaker to the output stage of a tube amplifier, providing a suitable totem-pole output-stage topology were employed [5]. Hence, the transformer, which has traditionally been regarded as the weak link of such amplifiers, could actually be used to enhance the low-frequency performance.

However, passive components, such as inductors and reversible electrolytic capacitors, are bulky and relatively expensive and, as such, are not really suitable for use in portable equipment. These problems can be solved simply by replacing the passive circuit of Fig. 2 with the equivalent active scheme of Fig. 3. Such a scheme can be implemented with either active analogue circuits (discrete or ASIC) or by digital filters. The latter would also allow the use of a digital class D amplifier.

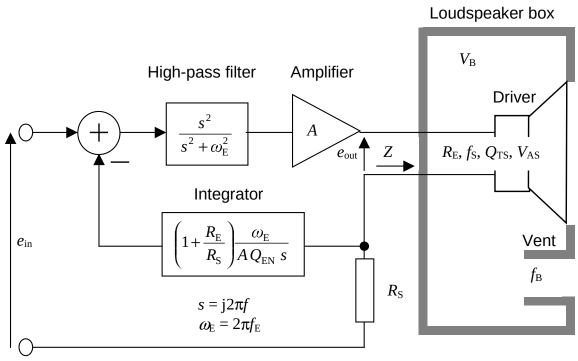

Fig. 3. Vented-box system with active 2nd-order non-isolated filter

The analogy of the LC filter is fairly intuitive. If the passive components in Fig. 2 have no losses, then the Q of the filter is infinite when there is no load connected. Current drawn by the load damps the resonance of the filter. Hence, the Q of the filter depends upon the load impedance Z. In this case, the load is the boxed driver, the impedance of which varies with frequency. The open loop Q of the filter in Fig. 3 is also infinite. Hence, its Q is determined entirely by the negative current-feedback loop and is therefore load dependent.

The current-feedback is derived from the current-sensing resistor R_S in series with the driver. The integrator in the feedback path ensures that the closed-loop output of the filter has the correct 90° phase lead and gain of $A \times Q_{EN}$ at the filter's resonance, at which point the open-loop gain is infinite. If $R_S \ll R_E$, the transfer function of the filter is

$$\frac{e_{out}}{e_{in}} = \frac{A s^2}{s^2 + \frac{R_E}{Z}\frac{\omega_E}{Q_{EN}} s + \omega_E^2} \quad (1)$$

$$= \frac{s^2}{s^2 + \frac{1}{ZC} s + \frac{1}{LC}} \quad (2)$$

if

$$\frac{\omega_E}{Q_{EN}} = \frac{1}{R_E C} \quad (3)$$

and

$$\omega_E^2 = \frac{1}{LC} \quad (4)$$

Hence, the transfer function of this active version is the same as that for the passive LC filter, except for some attenuation due to R_S. Obviously, R_S should be minimised, typically no greater than $0.15\,R_E$. In any case, passive components would also exhibit some losses.

There is just one important difference between the passive scheme of Fig. 2 and the active scheme of Fig. 3 in such cases where some power lift is required by the filter. The passive scheme does this by dropping the impedance presented to the amplifier's output terminals and drawing more current at the same voltage, whereas the active scheme lifts the voltage at the amplifier's output resulting in a smaller increase in current but at a higher voltage. The electrical impedance of the driver together with the amplifier's voltage and current drive capabilities should all be chosen with this in mind.

1.3 2nd-Order LEMF (many new solutions)

An ideal scenario would be one whereby we could tune the filter to give us a continuous range of solutions varying between the solutions for an isolated filter and those for a non-isolated one. This can be achieved by modifying the non-isolated filter scheme of Fig. 3 slightly to produce that of Fig. 4. The only difference is that in Fig. 3 the Q of the filter's transfer function is infinite, whereas in Fig. 4 it is Q_{EI}. We now have a parameter Q_{EI} that can be varied between infinity and the value defined for an isolated filter. In the case of the latter, Q_{EN} becomes infinite; thus removing the current-feedback loop and leaving behind an isolated filter as in Fig. 1. Varying Q_{EI} thus gives us the missing "in-between" solutions. These are shown in Table 2 below.

The parameters R_E, f_S, Q_{TS} and V_{AS} are standard parameters that are often supplied with the driver or can be measured. Given the enclosure size V_B, one simply has to find a ratio V_{AS}/V_B in the alignment table that fits the loudspeaker system. If no match can be found, then a new table could be generated for a different frequency-response shape, such as a Chebyshev response, using the formulae given in section 1.5 together with the root loci.

The value of Q_{TS} specified in the table may not be that of the chosen driver. However, it may be modified by the use of current-feedback around the amplifier taken from the current sensing resistor R_S.

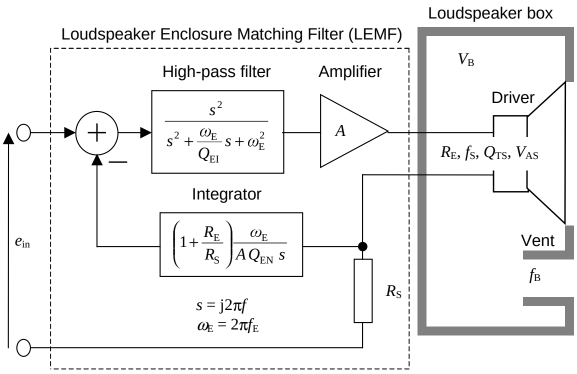

Fig. 4: Vented-box system with 2nd-order LEMF

Negative current-feedback increases Q_{TS} and positive current-feedback reduces it. Otherwise, a different frequency-response shape could be used. For example, a Chebyshev response would require a higher Q_{TS}.

Suggested active schemes for a 2nd-order LEMF are shown in Figs. 5 & 6. The scheme shown in Fig. 5 uses negative current-feedback to increase Q_{ES}. This is effectively achieved by using the feedback to increase the output impedance of the amplifier which is then added to R_E to define a new Q_{ES} value, denoted by Q'_{ES}.

Q_{EI}	Type 1 B6 LEMF alignments						Type 2 B6 LEMF alignments						Type 3 B6 LEMF alignments					
	f_3/f_S	V_{AS}/V_B	Q_{TS}	Q_{EN}	f_E/f_S	f_B/f_S	f_3/f_S	V_{AS}/V_B	Q_{TS}	Q_{EN}	f_E/f_S	f_B/f_S	f_3/f_S	V_{AS}/V_B	Q_{TS}	Q_{EN}	f_E/f_S	f_B/f_S
Inf.	Type 1 B6 Alignment with Non-Isolated Filter						Type 2 B6 Alignment with Non-Isolated Filter						Type 3 B6 Alignment with Non-Isolated Filter					
	0.487	0.379	0.607	0.820	0.194	0.596	1.000	4.464	0.282	3.157	1.000	1.000	2.052	1.067	0.607	0.820	5.153	1.677
10.00	0.495	0.389	0.601	0.883	0.201	0.604	1.000	4.112	0.284	4.100	1.000	1.000	2.021	1.067	0.601	0.883	4.986	1.657
8.000	0.497	0.391	0.599	0.900	0.202	0.606	1.000	4.025	0.285	4.418	1.000	1.000	2.014	1.067	0.599	0.900	4.943	1.651
7.000	0.498	0.393	0.598	0.913	0.204	0.607	1.000	3.963	0.285	4.673	1.000	1.000	2.008	1.067	0.598	0.913	4.913	1.648
5.000	0.503	0.399	0.594	0.955	0.208	0.611	1.000	3.766	0.287	5.699	1.000	1.000	1.990	1.067	0.594	0.955	4.815	1.636
4.000	0.507	0.404	0.591	0.996	0.212	0.615	1.000	3.596	0.288	6.982	1.000	1.000	1.973	1.067	0.591	0.996	4.728	1.625
3.000	0.514	0.414	0.585	1.071	0.218	0.623	1.000	3.318	0.291	10.77	1.000	1.000	1.945	1.067	0.585	1.071	4.580	1.606
2.500	0.520	0.421	0.580	1.139	0.224	0.628	1.000	3.101	0.293	17.84	1.000	1.000	1.922	1.067	0.580	1.139	4.460	1.591
2.000	0.530	0.434	0.573	1.257	0.234	0.638	1.000	2.786	0.298	131.5	1.000	1.000	1.885	1.067	0.573	1.257	4.275	1.567
1.932	0.532	0.437	0.571	1.280	0.236	0.640	Original Class I B6 Alignment with Isolated Filter						1.879	1.067	0.571	1.280	4.241	1.563
							1.000	2.732	0.299	Inf.	1.000	1.000						
1.500	0.549	0.458	0.560	1.515	0.253	0.656							1.820	1.066	0.560	1.515	3.953	1.525
0.900	0.622	0.545	0.520	3.081	0.335	0.719							1.607	1.053	0.520	3.081	2.985	1.390
0.800	0.657	0.583	0.506	4.329	0.380	0.748							1.521	1.042	0.506	4.329	2.633	1.337
0.750	0.684	0.610	0.497	5.592	0.416	0.769							1.462	1.032	0.497	5.592	2.404	1.300
0.707	0.716	0.641	0.487	7.589	0.462	0.794	Original Class II B6 Alignment with Isolated Filter						1.397	1.017	0.487	7.589	2.163	1.259
							1.000	1.000	0.408	Inf.	1.000	1.000						
0.700	0.723	0.647	0.486	8.075	0.472	0.799	1.000	0.988	0.411	460.2	1.000	1.000	1.384	1.013	0.486	8.075	2.118	1.251
0.650	0.787	0.702	0.472	14.66	0.576	0.848	1.000	0.910	0.433	65.68	1.000	1.000	1.270	0.977	0.472	14.66	1.737	1.180
0.620	0.863	0.757	0.463	27.26	0.712	0.903	1.000	0.866	0.448	49.96	1.000	1.000	1.159	0.929	0.463	27.26	1.405	1.108
0.610	0.912	0.789	0.460	37.14	0.808	0.938	1.000	0.852	0.454	47.91	1.000	1.000	1.097	0.897	0.460	37.14	1.238	1.066
0.600							1.000	0.838	0.460	47.00	1.000	1.000						
0.550							1.000	0.772	0.492	68.28	1.000	1.000						
0.518							Original Class III B6 Alignment with Isolated Filter											
							1.000	0.732	0.518	Inf.	1.000	1.000						

Table 2. Butterworth (B6) alignments for loss-less vented-box system with 2nd-order LEMF

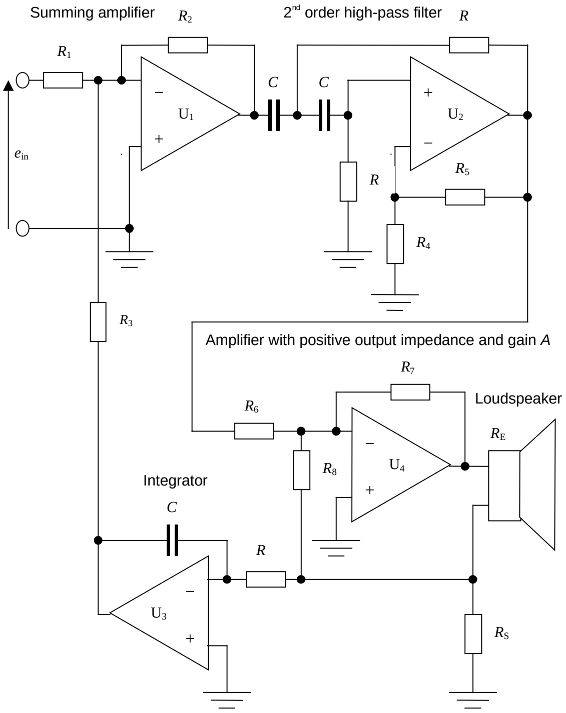

Fig. 5. Suggested active implementation of a 2nd-order LEMF in cases where the Q of the driver needs to be increased

Choose appropriate values for A, R_1, R_4, R_6, R_S & C bearing in mind that R_1 & R_6 are the load impedances presented to previous stages. G is the pass-band gain between the input and the driver.

$$R = \frac{1}{2\pi f_E C} \tag{5}$$

$$R_2 = \frac{G R_1}{A\left(3 - \dfrac{1}{Q_{EI}}\right)} \tag{6}$$

$$R_3 = \frac{G Q_{EN} R_1}{1 + \dfrac{R_E}{R_S}} \tag{7}$$

$$R_5 = \left(2 - \frac{1}{Q_{EI}}\right) R_4 \tag{8}$$

$$R_7 = A \frac{Q'_{ES}}{Q_{ES}} R_6 \tag{9}$$

$$R_8 = \frac{R_7}{\left(\dfrac{Q'_{ES}}{Q_{ES}} - 1\right)\dfrac{R_E}{R_S} - 1} \tag{10}$$

The scheme shown in Fig. 6 uses positive current-feedback to reduce Q_{ES}. This is effectively achieved by using the feedback to provide the amplifier with a negative output-impedance [6] which is then subtracted from R_E to define a new Q_{ES} value, denoted by Q'_{ES}. The amount of positive feedback needs to be applied judiciously since it will exaggerate the effects of variation in R_E due to tolerance as well as any power compression that results from variation of R_E with temperature. Negative current-feedback generally swamps variations in R_E. A more powerful magnet could be employed to minimise Q_{ES} in the first instance.

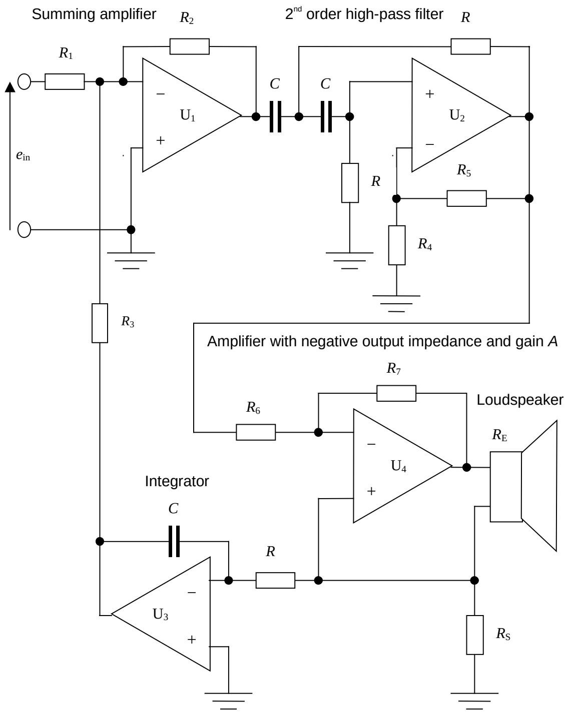

Fig. 6. Suggested active implementation of a 2nd-order LEMF in cases where the Q of the driver needs to be decreased

Choose appropriate values for R_1, R_4, R_6, R_S & C bearing in mind that R_1 & R_6 are the load impedances presented to previous stages. G is the pass-band gain between the input and the driver.

$$A = \left(1 - \frac{Q_{ES}}{Q'_{ES}}\right)\frac{R_E}{R_S} \tag{11}$$

$$R = \frac{1}{2\pi f_E C} \tag{12}$$

$$R_2 = \frac{G R_1}{A\left(3 - \dfrac{1}{Q_{EI}}\right)} \tag{13}$$

$$R_3 = \frac{G Q_{EN} R_1}{1 + \dfrac{R_E}{R_S}} \tag{14}$$

$$R_5 = \left(2 - \frac{1}{Q_{EI}}\right) R_4 \tag{15}$$

$$R_7 = \left(1 - \frac{Q'_{ES}}{Q_{ES}}\right)\frac{R_E}{R_S} R_6 \tag{16}$$

1.4 Vent Dimension Calculations

Although much has already been written about vent dimensions [7], some basic formulae are included here just for completeness. The box resonant-frequency f_B is the frequency at which the acoustic mass M_{AP} of the plug of air contained within the vent (usually consisting of a cylindrical hollow tube) resonates with the compliance C_{AB} of the air contained within the volume of the box. Given the box resonant-frequency f_B (in Hz), the box volume V_B (in m^3) and vent radius a_P (in m), then the length of the vent l_P (in m) can be calculated using the formula

$$l_P = \frac{c^2\, a_P^2}{4\pi\, f_B^2\, V_B} - k_E\, a_P \quad (17)$$

Alternatively, the radius can be calculated for a given length using the formula

$$a_P = \frac{2\pi\, k_E\, f_B^2\, V_B}{c^2}\left(1 + \sqrt{1 + \frac{c^2\, l_P}{\pi\, k_E^2\, f_B^2\, V_B}}\right) \quad (18)$$

where; c = speed of sound in air = 345 m/s
at T = 25°C and $P_0 = 10^5$ N/m^2

The two formulae above are based on the assumption that the effective length of the vent is $l_P + k_E\, a_P$ where k_E is the end correction factor.

If the tube is free at one end and mounted in a baffle at the other;

k_E = 1.46

If the tube is mounted in a baffle at both ends;

k_E = 1.7

The inside of the box can usually be regarded as a baffle termination, even if the tube stands proud of the inner wall. The outside termination is a baffle if the box is close to a boundary or free if the box is in free space. As can be seen, there is a certain degree of freedom regarding the vent size. For a given box resonant-frequency, the vent can be made short and narrow or long and wide. To avoid air turbulence or the possibility of the air mass popping out of the tube completely at high sound pressure levels, it is best to make the vent as long and wide as possible within the mechanical constraints of the design.

1.5 Transfer function of a vented-box system with a 2nd-order LEMF

A simplified equivalent electrical circuit of the driver in a vented box is shown in Fig. 7. All enclosure losses together with the driver mechanical loss are ignored. The driver coil inductance is also ignored. The acoustic mass of the air load is included with the diaphragm mass M_{MD} and also with the end correction of the vent mass M_{AP}. The sound pressure $p(r)$ at a distance r in free space for a given output volume velocity U_O is given by

$$p(r) = \frac{\rho_0\, s\, U_O}{4\pi\, r} \quad (19)$$

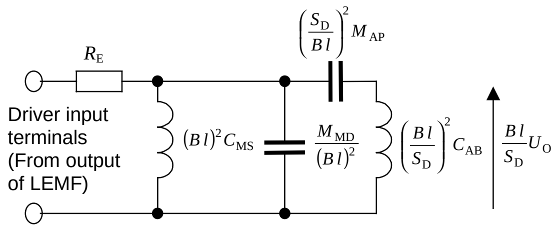

Fig. 7. Equivalent electrical circuit of a driver in a vented box

A generic 6th-order high-pass system transfer-function that relates the sound pressure $p(r)$ at a distance r in free space for a given voltage e_{in} at the input of the LEMF is given by

$$p(r) = \frac{\left(\frac{A\,\rho_0\, S_D\, B\, l}{4\pi\, r\, M_{MD}\,(R_E + R_S)}\right) s^6 e_{in}}{\left(s^2 + \frac{\omega_1}{Q_1} s + \omega_1^2\right)\left(s^2 + \frac{\omega_2}{Q_2} s + \omega_2^2\right)\left(s^2 + \frac{\omega_3}{Q_3} s + \omega_3^2\right)} \quad (20)$$

$$= \frac{\left(\frac{A\,\rho_0\, S_D\, B\, l}{4\pi\, r\, M_{MD}\,(R_E + R_S)}\right) s^6\, e_{in}}{s^6 + k_5\, s^5 + k_4\, s^4 + k_3\, s^3 + k_2\, s^2 + k_1\, s + k_0} \quad (21)$$

where

ρ_0 is the density of air (=1.18 kg/m^3 at T=22 °C and $P_0 = 10^5$ N/m^2),
S_D is the effective surface area of the diaphragm (m^2),
B is the magnetic flux density in the air gap (T),
l is the length of voice coil conductor in magnetic gap (m)

$$k_0 = \omega_1^2\, \omega_2^2\, \omega_3^2 \quad (22)$$

$$k_1 = \omega_1\, \omega_2\, \omega_3 \left(\frac{\omega_1\, \omega_2}{Q_3} + \frac{\omega_1\, \omega_3}{Q_2} + \frac{\omega_2\, \omega_3}{Q_1}\right) \quad (23)$$

$$k_2 = \omega_1^2\, \omega_2^2 + \omega_1^2\, \omega_3^2 + \omega_2^2\, \omega_3^2 + \omega_1\, \omega_2\, \omega_3 \left(\frac{\omega_1}{Q_2 Q_3} + \frac{\omega_2}{Q_1 Q_3} + \frac{\omega_3}{Q_1 Q_2}\right) \quad (24)$$

$$k_3 = \frac{\omega_1\,(\omega_2^2 + \omega_3^2)}{Q_1} + \frac{\omega_2\,(\omega_1^2 + \omega_3^2)}{Q_2} + \frac{\omega_3\,(\omega_1^2 + \omega_2^2)}{Q_3} + \frac{\omega_1\, \omega_2\, \omega_3}{Q_1\, Q_2\, Q_3} \quad (25)$$

$$k_4 = \omega_1^2 + \omega_2^2 + \omega_3^2 + \frac{\omega_1\, \omega_2}{Q_1\, Q_2} + \frac{\omega_1\, \omega_3}{Q_1\, Q_3} + \frac{\omega_2\, \omega_3}{Q_2\, Q_3} \quad (26)$$

$$k_5 = \frac{\omega_1}{Q_1} + \frac{\omega_2}{Q_2} + \frac{\omega_3}{Q_3} \quad (27)$$

The denominator polynomial in s can be tailored to produce a standard filter frequency-response shape. For example, a Butterworth polynomial has

$$\omega_1 = \omega_2 = \omega_3 = 2\pi\, f_{3dB} \quad (28)$$

$$Q_1 = \frac{1}{2\ \mathrm{Cos}\ 15°} = \frac{\sqrt{2}}{\sqrt{3}+1} = 0.5176 \quad (29)$$

$$Q_2 = \frac{1}{2\ \mathrm{Cos}\ 45°} = \frac{1}{\sqrt{2}} = 0.7071 \quad (30)$$

$$Q_3 = \frac{1}{2\ \mathrm{Cos}\ 75°} = \frac{\sqrt{2}}{\sqrt{3}-1} = 1.9319 \quad (31)$$

In the complex plane, the poles (polynomial roots) lie on a circle with an angle of 30° between them.

The actual transfer function that relates the sound pressure $p(r)$ at a distance r in free space for a given voltage e_{in} at the input of the LEMF is given by

$$p(r)=\frac{\left(\dfrac{A\rho_0 S_D Bl}{4\pi r M_{MD}(R_E+R_S)}\right)s^6 e_{in}}{\left\{\begin{array}{l} s^6+\left[\dfrac{\omega_S}{Q_{ES}}+\dfrac{\omega_E}{Q_{EI}}+\dfrac{\omega_E}{Q_{EN}}\right]s^5+\left[\left(1+\dfrac{V_{AS}}{V_B}\right)\omega_S^2+\omega_B^2+\omega_E^2+\dfrac{\omega_S\omega_E}{Q_{ES}Q_{EI}}\right]s^4+\left[\dfrac{\omega_S}{Q_{ES}}\left(\omega_B^2+\omega_E^2\right)+\left(\dfrac{\omega_E}{Q_{EI}}+\dfrac{\omega_E}{Q_{EN}}\right)\left\{\left(1+\dfrac{V_{AS}}{V_B}\right)\omega_S^2+\omega_B^2\right\}\right]s^3 \\ +\left[\omega_S^2\omega_B^2+\left(1+\dfrac{V_{AS}}{V_B}\right)\omega_S^2\omega_E^2+\omega_B^2\omega_E^2+\dfrac{\omega_S\omega_B^2\omega_E}{Q_{ES}Q_{EI}}\right]s^2+\omega_S\omega_B\omega_E\left[\dfrac{\omega_B\omega_E}{Q_{ES}}+\omega_S\omega_B\left(\dfrac{1}{Q_{EI}}+\dfrac{1}{Q_{EN}}\right)\right]s+\omega_S^2\omega_B^2\omega_E^2 \end{array}\right\}} \quad (32)$$

where

$$\omega_S = 2\pi f_S = \frac{1}{\sqrt{M_{MD}C_{MS}}} \quad (33)$$

$$\omega_B = 2\pi f_B = \frac{1}{\sqrt{M_{AP}C_{AB}}} \quad (34)$$

$$Q_{ES} = \frac{R_E}{(Bl)^2}\sqrt{\frac{M_{MD}}{C_{MS}}} \quad (35)$$

$$\frac{V_{AS}}{V_B} = \frac{S_D^2 C_{MS}}{C_{AB}} \quad (36)$$

Eq. (32) tells us that a sensitive loudspeaker is one that has a powerful magnet and a large diaphragm of low mass. If this is so, then the suspension compliance (or V_{AS}) has to be high in order to achieve an extended low-frequency response (low f_s). Therefore, for a given compliance ratio, the box volume V_B also has to be large. In practice, compromises usually have to be made.

Equating the polynomial coefficients of the two transfer functions, given by Eqs. (21) & (32), yields a set of six simultaneous equations. Solving these equations gives us the six following Eqs. (37) to (42) that are used to generate each alignment from the root loci

$$\begin{array}{l} Q_{EI}^2\omega_E^{18}-k_4Q_{EI}^2\omega_E^{16}+k_3Q_{EI}\omega_E^{15}-(k_1+k_2k_5)Q_{EI}\omega_E^{13}+\left[k_1k_5+(k_2k_4-k_0)Q_{EI}^2\right]\omega_E^{12}+(k_0k_5-k_1k_4)Q_{EI}\omega_E^{11} \\ -\left[k_0k_5^2+(k_2^2-k_0k_4)Q_{EI}^2\right]\omega_E^{10}+2\left[k_0(k_4k_5-k_3)+k_1k_2\right]Q_{EI}\omega_E^9-\left[k_1^2+k_0(k_4^2-k_2)Q_{EI}^2\right]\omega_E^8+k_0(k_1-k_2k_5)Q_{EI}\omega_E^7 \\ +k_0\left[k_1k_5+(k_2k_4-k_0)Q_{EI}^2\right]\omega_E^6 - k_0(k_0k_5+k_1k_4)Q_{EI}\omega_E^5+k_0^2k_3Q_{EI}\omega_E^3-k_0^2k_2Q_{EI}^2\omega_E^2+k_0^3Q_{EI}^2=0 \end{array} \quad (37)$$

Eq. (37) above is solved for ω_E. Although this is an 18th-order polynomial with eighteen roots, only a maximum of three are positive and real.

$$\begin{array}{l} [(4Q_{EI}^2-3Q_{EI}^4-1)\omega_E^5+k_5Q_{EI}(Q_{EI}^4-3Q_{EI}^2+1)\omega_E^4+k_4Q_{EI}^2(2Q_{EI}^2-1)\omega_E^3+k_3Q_{EI}^3(1-Q_{EI}^2)\omega_E^2 - k_2Q_{EI}^4\omega_E+k_1Q_{EI}^5]Q_{EN}^3 \\ +[(8Q_{EI}^2-3Q_{EI}^4-3)\omega_E^4+k_5Q_{EI}(2-3Q_{EI}^2)\omega_E^3+2k_4Q_{EI}^2(Q_{EI}^2-1)\omega_E^2+k_3Q_{EI}^3\omega_E-k_2Q_{EI}^4]Q_{EI}\omega_EQ_{EN}^2 \\ +[(4Q_{EI}^2-3)\omega_E^2+k_5Q_{EI}\omega_E-k_4Q_{EI}^2]Q_{EI}^2\omega_E^3Q_{EN}-Q_{EI}^3\omega_E^5=0 \end{array} \quad (38)$$

Eq. (38) above is solved for Q_{EN}.

$$\omega_B=\sqrt{\frac{k_1-\left(\dfrac{1}{Q_{EI}}+\dfrac{1}{Q_{EN}}\right)\left\{k_2\omega_E-k_4\omega_E^3+\omega_E^5+\left[k_5-\left(\dfrac{1}{Q_{EI}}+\dfrac{1}{Q_{EN}}\right)\omega_E\right]\dfrac{\omega_E^4}{Q_{EI}}\right\}}{\left[1-\left(\dfrac{1}{Q_{EI}}+\dfrac{1}{Q_{EN}}\right)\dfrac{1}{Q_{EI}}\right]\left[k_5-\left(\dfrac{1}{Q_{EI}}+\dfrac{1}{Q_{EN}}\right)\omega_E\right]\omega_E^2}} \quad (39)$$

$$\frac{V_{AS}}{V_B}=\frac{-k_2-\omega_B^4+k_4\left(\omega_B^2+\omega_E^2\right)-\omega_B^2\omega_E^2-\omega_E^4-\left[k_5-\left(\dfrac{1}{Q_{EI}}+\dfrac{1}{Q_{EN}}\right)\omega_E\right]\dfrac{\omega_E^3}{Q_{EI}}}{k_2-k_4\omega_E^2+\omega_E^4+\left(\omega_E^2-\omega_B^2\right)\left[k_5-\left(\dfrac{1}{Q_{EI}}+\dfrac{1}{Q_{EN}}\right)\omega_E\right]\dfrac{\omega_E}{Q_{EI}}} \quad (40)$$

$$Q_{ES}=\frac{\sqrt{k_2-k_4\omega_E^2+\omega_E^4+\left(\omega_E^2-\omega_B^2\right)\left[k_5-\left(\dfrac{1}{Q_{EI}}+\dfrac{1}{Q_{EN}}\right)\omega_E\right]\dfrac{\omega_E}{Q_{EI}}}}{\left[k_5-\left(\dfrac{1}{Q_{EI}}+\dfrac{1}{Q_{EN}}\right)\omega_E\right]\omega_B} \quad (41)$$

$$\omega_S=\left[k_5-\left(\frac{1}{Q_{EI}}+\frac{1}{Q_{EN}}\right)\omega_E\right]Q_{ES} \quad (42)$$

2 1ST-ORDER FILTERS

2.1 1st-Order Isolated Filter (1 solution)

According to A.N. Thiele [2], the configuration shown in Fig. 8 has just one solution for any given frequency-response shape. For a Butterworth response, the value of V_{AS}/V_B is unity and the cut-off frequency is equal to f_s. Such a filter is easy to implement. For example, a simple passive RC filter could be used with

$$C = \frac{1}{2\pi\ f_E\ R} \tag{43}$$

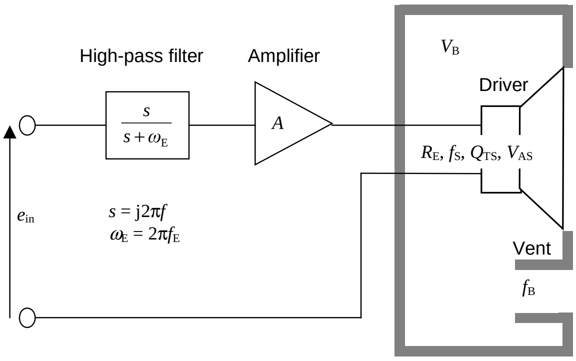

Fig. 8. Vented-box system with 1st-order isolated filter

2.2 1st-Order Non-Isolated Filter (1 solution)

If we place a passive 1st-order filter between the amplifier and loudspeaker as shown in Fig. 9, the filter interacts with the loudspeaker's input impedance to produce a new solution, for any particular frequency-response shape, that is different from the isolated filter solution. In the case of the Butterworth solution, the value of V_{AS}/V_B becomes 0.528 and the cut-off frequency is extended down to 0.618 f_s. This gives us just over 2/3 octave of extra low-frequency extension.

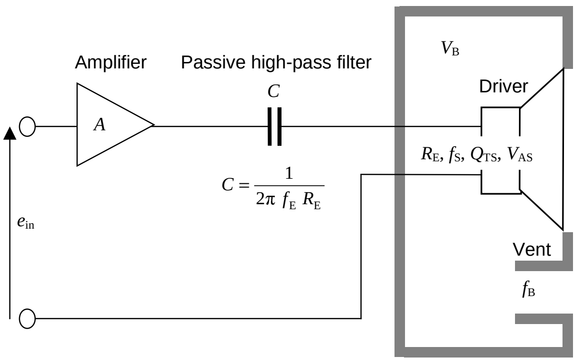

Fig. 9. Vented-box system with passive 1st-order non-isolated filter

However, whilst such a passive filter could be useful for tailoring the low-frequency response of a passive hi-fi loudspeaker, the reversible electrolytic capacitor needed is a bulky and relatively expensive item and, as such, is not really suitable for use in portable equipment. These problems can be solved simply by replacing the passive circuit of Fig. 9 with the equivalent active scheme of Fig. 10. Such a scheme can be implemented with either active analogue circuits (discrete or ASIC) or by digital filters. The latter would also allow the use of a digital class D amplifier.

The analogy of the passive filter is fairly intuitive. The capacitor in Fig. 9 exhibits an impedance that increases as frequency decreases. Hence it reduces the amplitude of the low frequencies. However, the degree of attenuation depends upon the impedance of the loudspeaker, which varies somewhat over the frequency range. Therefore, there is interaction between the filter and the loudspeaker. In Fig. 10, the low-frequency attenuation is produced by negative feedback that increases as frequency decreases. Because the feedback is derived from the current-sensing resistor R_S, it will vary according to the impedance of the loudspeaker. Therefore, this configuration will also result in interaction between the loudspeaker and the filter.

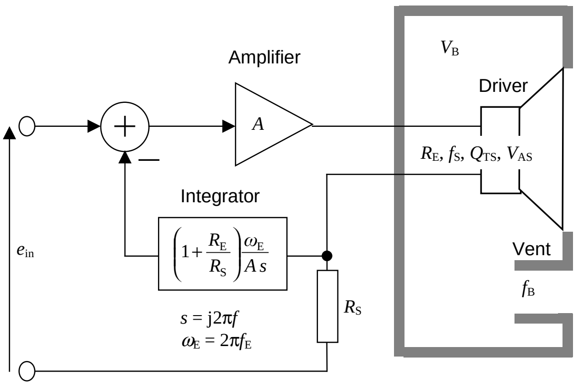

Fig. 10. Vented-box system with active 1st-order non-isolated filter

2.3 1st-Order LEMF (many new solutions)

The isolated Butterworth alignment results in an enclosure volume V_B that is equal to V_{AS} and f_3 equal to f_S. On the other hand, the non-isolated Butterworth alignment results in an enclosure volume that is equal to 1.89 V_{AS} and f_3 equal to 0.618 f_S (an extra 2/3 octave). It would be rather useful if a range of Butterworth alignments in between these two extremes could be utilised.

This can be achieved using the circuit shown in Fig. 11 below. It comprises the same non-isolated filter as the one in Fig. 10 but this time preceded by an isolated 1st-order filter network with a zero or "shelf frequency" at ω_Z. The initial roll-off of the filter starts at around ω_F, but levels off at around ω_Z. At this point, the non-isolated filter takes over so that the roll-off is seamless. In other words, at ω_Z the pole of the non-isolated filter cancels the zero of the isolated filter.

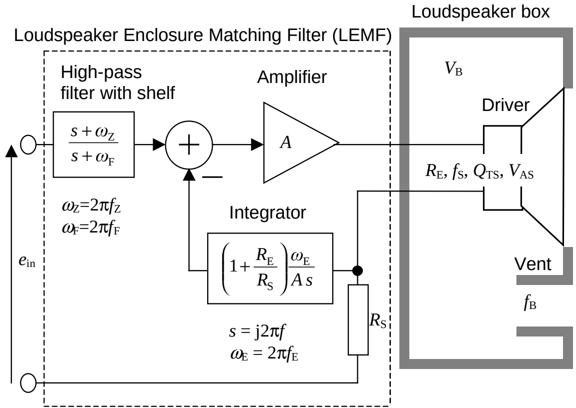

Fig. 11. Vented-box system with 1st-order LEMF

By varying the shelf frequency ω_Z from dc up to ω_F, a range of solutions is obtained from isolated (ω_Z=0) to non-isolated ($\omega_Z=\omega_F$).

2.4 Transfer function of a vented-box system with a 1st-order LEMF

The actual transfer function that relates the sound pressure $p(r)$ at a distance r in free space for a given voltage e_{in} at the input of the LEMF is given by

$$p(r)=\frac{\left(\frac{s+\omega_Z}{s+\omega_F}\right)\left(\frac{\rho_0\, S_D\, Bl}{4\pi r\, M_{MD}\,(R_E+R_S)}\right)s^5\, e_{in}}{s^5+\left(\frac{\omega_S}{Q_{ES}}+\omega_E\right)s^4+\left\{\left(1+\frac{V_{AS}}{V_B}\right)\omega_S^2+\omega_B^2\right\}s^3+\left[\frac{\omega_S\,\omega_B^2}{Q_{ES}}+\left\{\left(1+\frac{V_{AS}}{V_B}\right)\omega_S^2+\omega_B^2\right\}\omega_E\right]s^2+\omega_S^2\,\omega_B^2\, s+\omega_S^2\,\omega_B^2\,\omega_E} \quad (44)$$

A generic 6th-order high-pass system transfer-function that relates the sound pressure $p(r)$ at a distance r in free space for a given voltage e_{in} at the input of the LEMF is given by

$$p(r)=\frac{\left(\frac{s+\omega_Z}{s+\omega_3}\right)\left(\frac{\rho_0\, S_D\, Bl}{4\pi r\, M_{MD}\,(R_E+R_S)}\right)s^5\, e_{in}}{\left(s^2+\frac{\omega_1}{Q_1}s+\omega_1^2\right)\left(s^2+\frac{\omega_2}{Q_2}s+\omega_2^2\right)(s+\omega_Z)} \quad (45)$$

It can be seen that the $(s+\omega_Z)$ terms in the numerator and denominator cancel. Equating the polynomial coefficients of the two transfer functions, given by Eqs. (44) & (45), yields a set of five simultaneous equations. Solving these equations gives us the following five Eqs. (46) to (50) that are used to generate each alignment from the root loci

$$\omega_E=\frac{\omega_1\,\omega_2\,\omega_Z}{\omega_1\,\omega_2+\left(\frac{\omega_1}{Q_2}+\frac{\omega_2}{Q_1}\right)\omega_Z} \quad (46)$$

$$\omega_B=\sqrt{\frac{\omega_1\,\omega_2\left(\frac{\omega_1}{Q_2}+\frac{\omega_2}{Q_1}\right)-\omega_Z\,\omega_E\left(\frac{\omega_1}{Q_1}+\frac{\omega_2}{Q_2}\right)+(\omega_Z-\omega_E)\left(\omega_1^2+\omega_2^2+\frac{\omega_1\,\omega_2}{Q_1\,Q_2}\right)}{\frac{\omega_1}{Q_1}+\frac{\omega_2}{Q_2}+\omega_Z-\omega_E}} \quad (47)$$

$$\omega_S=\frac{\sqrt{\omega_1\,\omega_2\left[\omega_1\,\omega_2+\left(\frac{\omega_1}{Q_2}+\frac{\omega_2}{Q_1}\right)\omega_Z\right]}}{\omega_B} \quad (48)$$

$$Q_{ES}=\frac{\omega_S}{\frac{\omega_1}{Q_1}+\frac{\omega_2}{Q_2}+\omega_Z-\omega_E} \quad (49)$$

$$\frac{V_{AS}}{V_B}=\frac{1}{\omega_S^2}\left[\omega_1^2+\omega_2^2+\frac{\omega_1\,\omega_2}{Q_1\,Q_2}+\omega_Z\left(\frac{\omega_1}{Q_1}+\frac{\omega_2}{Q_2}\right)-\left(\omega_S^2+\omega_B^2\right)\right] \quad (50)$$

These are essentially the same equations as for the non-isolated filter but with ω_3 replaced by ω_Z where;

$$0\le\omega_Z\le\omega_3 \quad (51)$$

and

$$\omega_F=\omega_3 \quad (52)$$

Hence

$$0\le\frac{\omega_Z}{\omega_F}\le 1 \quad (53)$$

The last Eq. (53) forms a parameter that becomes the basis of an alignment chart for a 1st-order LEMF as shown in Table 3

f_Z/f_F	f_3/f_S	V_{AS}/V_B	Q_{TS}	f_Z/f_S	f_F/f_S	f_E/f_S	f_B/f_S
1.00	B5 Alignment with Non-Isolated Filter						
	0.618	0.528	0.553	0.618	0.618	0.191	0.687
0.80	0.652	0.533	0.558	0.522	0.652	0.187	0.711
0.60	0.696	0.553	0.557	0.417	0.696	0.178	0.741
0.40	0.754	0.603	0.547	0.302	0.754	0.159	0.783
0.30	0.793	0.648	0.535	0.238	0.793	0.142	0.813
0.20	0.842	0.717	0.517	0.168	0.842	0.116	0.854
0.15	0.872	0.765	0.504	0.131	0.872	0.098	0.880
0.10	0.908	0.825	0.489	0.091	0.908	0.074	0.911
0.05	0.949	0.902	0.470	0.047	0.949	0.043	0.951
0.00	Original B5 Alignment with Isolated Filter						
	1.000	1.000	0.447	0.000	1.000	0.000	1.000

Table 3. Butterworth (B5) alignments for loss-less vented-box system with 1st-order LEMF

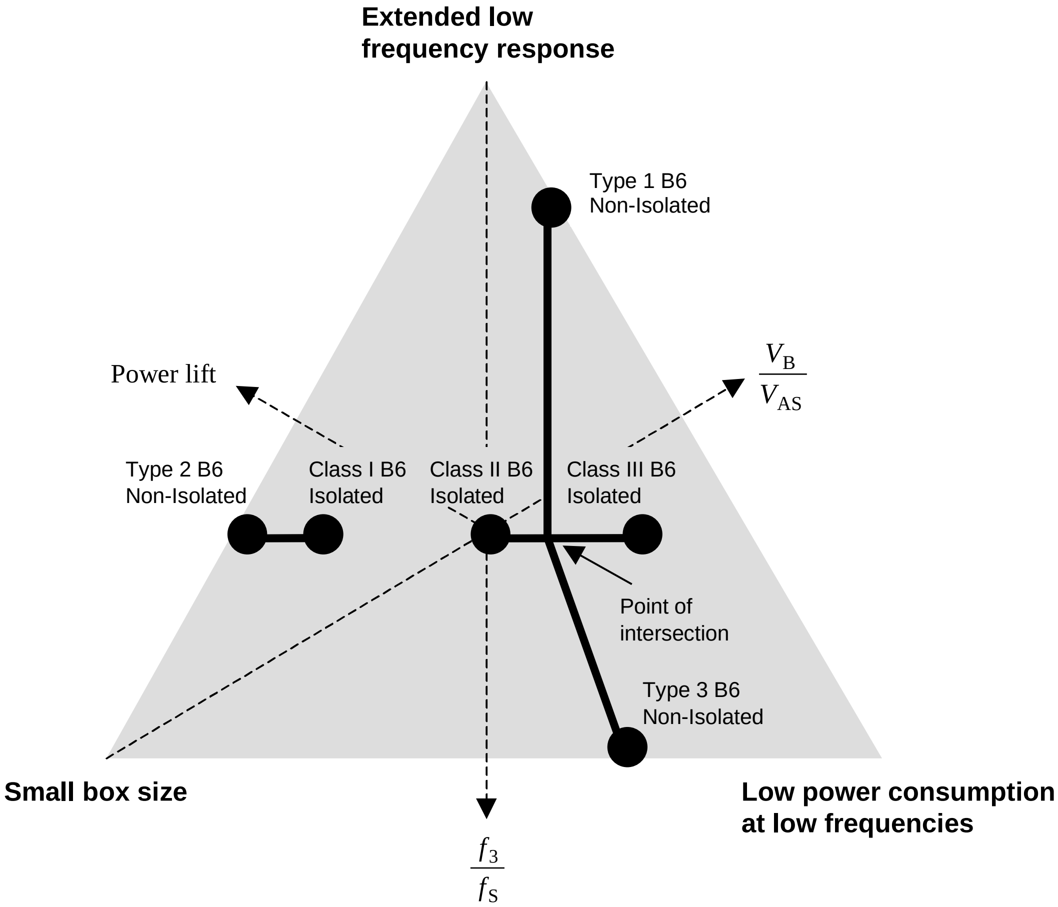

Figure 12. Loudspeaker triangle for B6 alignments

3 CONCLUSIONS

An effective method to tune a 5th or 6th-order vented-box loudspeaker system to produce a pre-determined frequency-response shape, over a fairly wide range of box volumes, has been described. As pointed out by R.H. Small [3], an inescapable fact of loudspeaker design is that there is a three-way efficiency/bandwidth/size trade-off. This relationship can be symbolised by the loudspeaker triangle in figure 12 above. It can be seen how the 2nd order LEMF helps us to choose the appropriate solution within the triangle for a particular design. The isolated and non-isolated filter alignments are shown by the large dots and the LEMF alignments are represented by the thick tracks that join them.

It is interesting to compare this method with the elegant ACE-bass system described by K.E. Ståhl [8], especially as the current-feedback loop forms a band-pass filter in both systems (with the mechanical circuit blocked). The ACE-bass system can be used to match a driver to a smaller box than that to which it would normally be suited using conventional alignments and can therefore perform a similar function to the Type 2 LEMF. It has the additional flexibility of allowing theoretically unlimited low-frequency extension, albeit by drawing yet more power from the amplifier.

However, the ACE-bass system cannot match the driver to a larger box, as does the Type 1 LEMF, which allows the low-frequency response to be extended with a more moderate increase in power. The reason for this is that the ACE-bass system reduces the effective compliance of the driver's suspension by placing another 'virtual' compliance in parallel with it. The cut-off frequency can then be extended downwards by adding virtual mass to the diaphragm. Re-adjustment of Q_{TS} is also allowed for.

By contrast, the LEMF allows use of either larger or smaller boxes, albeit within limits. Also, the design is achieved more directly through use of alignment tables and Thiele-Small parameters without any need to calculate individual driver parameters such as mass or compliance. Furthermore, the LEMF scheme is less complex because it simply involves applying current feedback via an integrator and uses the existing high-pass filter without the need for an additional band-pass filter. Of course, the ultimate in design flexibility could be achieved by combining the two systems.

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